Math 16a Review Problems

Monday, December 7, 2020 9:31 PM

Question 7

0 / 2 pts

Compute

$$\int_{-3}^{2} |x|\, dx$$

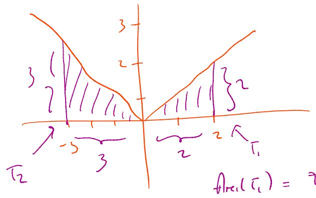
If the integral does not exist, enter 1000 as your answer.

2.5

6.5 (with margin: 0)

Integration

Always solve graphically if possible.



Area(T_1) = $(2)(2)\left(\frac{1}{2}\right) = 2$

Area(T_2) = $(3)(3)\left(\frac{1}{2}\right) = 4.5$

6.5

$$\int_{-3}^{2} |x|\,dx = \int_{-3}^{0} |x|\,dx + \int_{0}^{2} |x|\,dx$$

$$= \int_{-3}^{0} -x\,dx + \int_{0}^{2} x\,dx$$

Question 7

0 / 3 pts

What is the (y-coordinate of the) y-intercept of the tangent line to the curve defined by $(x^2+y^2)^2 = x^2+3xy+y^2+26$ at the point $(-1, 2)$?

LHS RHS

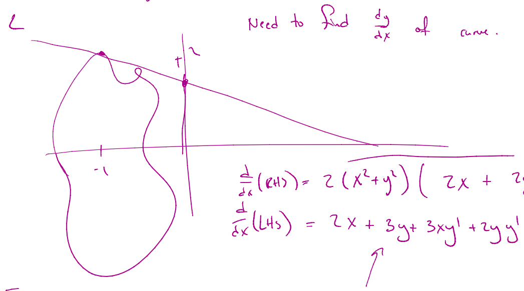
You may use a four-function calculator to compute the result. Present your answer to the tenths place.

1.1

2.6154 (with margin: 0.1)

implicit differentiation

Need to find $\frac{dy}{dx}$ of curve.



$\frac{d}{dx}(\text{RHS}) = 2(x^2+y^2)(2x+2yy')$

$\frac{d}{dx}(\text{LHS}) = 2x + 3y + 3xy' + 2yy'$

$y'(3x+2y) + 3y + 2x = 4y'y(x^2+y^2) + 4x(x^2+y^2)$

$y'(3x+2y+4y(x^2+y^2)) = 4x(x^2+y^2) - 3y + 2x$

$m \longrightarrow y' = \dfrac{4x(x^2+y^2) - 3y + 2x}{3x+2y+4y(x^2+y^2)}$

$x = -1$, $y = 2$ } plug in values.

~~Not actually~~

Tangent Line: $(y - y_1) = m(x - x_1)$

$(y - 2) = m(x+1)$

$y = 2 + m(x+1)$

y-int is when $x = 0$ for eq. of line.

$y(0) = 2 + m(0+1)$

$= 2 + m$

Question 4

0 / 3 pts

The functions $x(t)$ and $y(t)$ satisfy the following equation.

$$\frac{d}{dt}\left(x^2 + 4xy + 2y^2 + 2\right) = 0$$

Let $x' = \frac{dx}{dt}$, $y' = \frac{dy}{dt}$

Suppose that we know that $x(0) = -2$, $x(7) = 2$, $\frac{dx}{dt}(0) = 4$, $\frac{dx}{dt}(7) = -5$, $y(0) = 3$, and $y(7) = -1$

.

What is $\frac{dy}{dt}(0)$?

25

-8 (with margin: 0)

implicit differentiation

$2xx' + 4(x'y + xy') + 4yy' = 0$

$x = -2$, $x' = 4$, $y = 3$

Question 2

0 / 4 pts

For which of the following functions $f(x)$ is $f(\sin(x))$ continuous on $(-\infty, \infty)$?

- $\tan(\pi x)$ — $x = 1/2$ bad!
- $\frac{1}{x}$ — $x = 0$ bad!
- $f(x) = \frac{1}{x-2}$ — bad @ $x = 2$
- $\ln(x)$ — bad $x < 0$
- None of these

$\frac{1}{\sin(x) - 2}$ is continuous

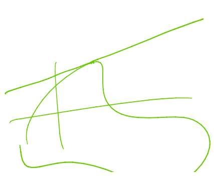
continuity, domains of functions

15 (with margin: 0)

Question 5

If x and y are related by the equation

$\ln(x - y) + 2y = x^2 - 2$



If x and y are related by the equation

$\boxed{\ln(x-y)+2y=x^2-2}$

and that have the values

$x=2$, $y=1$, and $dx=0.01$.

What is dy?

Present your answer to the hundredths place.

$y=f(x)$

$\frac{dy}{dx}=f'(x)$

$\boxed{dy=f'(x)\,dx}$

$f'(x)=\frac{dy}{dx}$

you find this by implicit dif.

linear approximation

Question 7 0 / 3 pts

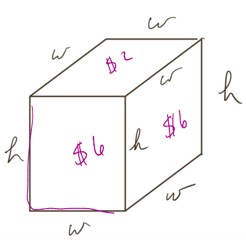
What is the (y-coordinate of the) y-intercept of the tangent line to the curve defined by $(x^2-y^2)^2=x^2+3xy+y^2+10$ at the point $(-1,2)$?

You may use a four-function calculator to compute the result. Present your answer to the tenths place.

Question 3 0 / 3 pts

A closed box with a square base, of dimensions $w \times w$ feet and height of h feet, is to have a total volume of 9 cubic feet. The material for this box costs \$2 per square foot for the top and bottom and \$6 per square foot for the sides.

What is the minimal cost in dollars of such a box?



$C = \$2\cdot(\underbrace{\text{area of top \& bot.}}_{a_1}) + \$6\cdot(\underbrace{\text{area of sides}}_{a_2})$

$a_1 = w^2+w^2 = 2w^2$

$a_2 = 2wh + 2wh = 4wh$

$C = 2(2w^2) + 6(4wh)$

$= 4w^2 + 24wh$

Need constraint

$9 = \text{Volume} = w\cdot w\cdot h = w^2h$

$\Rightarrow h = \frac{9}{w^2}$

Sub into Cost fxn

$C = 4w^2 + 24w\left(\frac{9}{w^2}\right) = 4w^2 + \frac{216}{w}$

$C'(w) = 8w - \frac{216}{w^2} = 0$

$8w - \frac{24\cdot 9}{w^2} = 0$ $\qquad w \neq 0$

$8w^3 - 24\cdot 9 = 0$

$w^3 = \frac{24\cdot 9}{8} = \frac{3\cdot\cancel{8}\cdot 9}{\cancel{8}} = 27$

$w = \sqrt[3]{27} = 3$ $\qquad h = \frac{9}{w^2} = \frac{9}{9} = 1$ $\qquad C(w,h) = 4\cdot 9 + 24\cdot 3\cdot 1 = 36 + \$72 = 108$

$2

optimization

What is the absolute maximal **value** of the function $f(x) = 2x^3 - 3x^2 - 36x + 4$ over the interval $[-3,0]$?

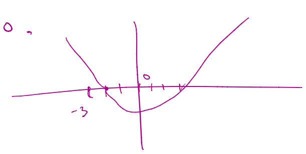
(1) $f'(x) = 6x^2 - 6x - 36 = 0$

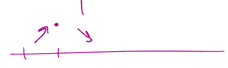
$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\Rightarrow x = -2, 3$

throw away.





maximization

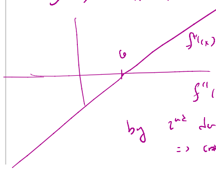
The functions $x(t)$ and $y(t)$ satisfy the following equation.

$2x^2+2xy+y^2=5$

Suppose that we know that $x(0)=-2$, $x(7)=2$, $\frac{dx}{dt}(0)=4$, $\frac{dx}{dt}(7)=-5$, $y(0)=3$, and $y(7)=-1$.

What is $\frac{dy}{dt}(7)$?

$f''(x) = 12x - 6$



$f''(x) < 0$ if $x \in (-3,0]$

by 2nd deriv. test

$\Rightarrow$ rel. max @ $x=-2$

$f(-3) =$ $\quad f(0) = 4$

$f(-2)$

implicit differenationa

Let $f(x)=x^5-3x+1$.

Which of the following is an approximate solution to $f(x)=0$ obtained using the method of linear approximation with an initial guess of $x=0$?

linear approximation

What is the (y-coordinate of the) y-intercept of the tangent line to the curve defined by $(x^2+y^2)^2=x^2+3xy+y^2+26$ at the point $(-1,2)$?

You may use a four-function calculator to compute the result. Present your answer to the tenths place.

What is the (y-coordinate of the) y-intercept of the tangent line to the curve defined by $(x^2 + y^2)^2 = x^2 + 3xy + y^2 + 26$ at the point $(-1, 2)$?

You may use a four-function calculator to compute the result. Present your answer to the tenths place.

At what value of x does the graph of $y = f(x) = x^9 e^{-\frac{1}{2}x^2}$ for $x > 0$ have a relative maximum?

maximization